Automatic positioning of landmarks for shape analysis

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This presentation

Introduction

2D

3D

Ending
Shape analysis

Contributions in for example medical image analysis:

- speed
- accuracy
- backup check

Automatic positioning of landmarks for shape analysis – p.3/46
Active shape

\[
shape = meanshape + \sum b_i \text{mode}_i
\]

\[
s = m + Mb
\]
Shape representation

- A set of landmarks on the curve/surface.
Shape representation

- A set of landmarks on the curve/surface.
- Correspondences across the training set of shapes.
Shape representation

• A set of landmarks on the curve/surface.
• Correspondences across the training set of shapes.
• Modes for Active Shape model obtained by PCA on covariance matrix of landmark coordinates.
Principal Component Analysis, PCA

Calculate orthogonal principal components and the variances in these components.
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Calculate eigenvectors (principal components) and eigenvalues (variances) of covariance matrix of landmark coordinates.
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Each column in $S$ has the deviations from the mean for one shape.

$$C_1 = \frac{1}{s}SS^T$$
Principal Component Analysis, PCA

Calculate orthogonal principal components and the variances in these components.

Calculate eigenvectors (principal components) and eigenvalues (variances) of covariance matrix of landmark coordinates.

Each column in $S$ has the deviations from the mean for one shape.

$$C_1 = \frac{1}{s} SS^T$$

$$C_2 = \frac{1}{s} S^T S$$
Landmark placement

- By hand
- Equally spaced
- Automatically and optimally
2D

Open curves

Establishing correspondences

Results

back
Open curves
Establishing correspondences

Goal  Find parameterizations so that \( \{curve_i(t)\} \) are corresponding points across the training set for all \( t \).

Method  Optimize parameterizations of the curves using Minimum Description Length as evaluation function.

Reparameterizations
Evaluation
Determining optimal reparameterizations
Stability
Reparameterization

- Place control points.
- Interpolate.
Evaluating a set of landmarks

- Align shapes.
Evaluating a set of landmarks

- Align shapes.
- Build model.
Evaluating a set of landmarks

- Align shapes.
- Build model.
- Calculate MDL using different numbers of modes.
Evaluating a set of landmarks

- Align shapes.
- Build model.
- Calculate MDL using different numbers of modes.
- Pick the best one.
MDL

- MDL is the minimum amount of data needed to code the model and the examples.
MDL

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- Minimizing MDL should give the simplest model that still represents the examples well.
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- Minimizing MDL should give the simplest model that still represents the examples well.
- The simplest model should generalize and capture the relevant modes of variation best.
Evaluation function

The full MDL is complex.
Use variable part of lower bound.

\[
f(m) = (nc+ns) \sum_{k=1}^{m} \log \lambda_k + (nc(ns-1) - m(nc+ns)) \log \left( \frac{\sum_{k=m+1}^{nc} \lambda_k}{nc} \right)
\]

\[
\approx (nc+ns) \sum_{k=1}^{m} \log(\lambda_k + \varepsilon) + (nc(ns-1) - m(nc+ns)) \log \left( \frac{\sum_{k=m+1}^{nc} \lambda_k}{nc} + \varepsilon \right)
\]
Determining parameterizations

- Place control point 1 between 0 and 1 on all curves. Optimize its positions using f(m) as evaluation.
Determining parameterizations

- Place control point 1 between 0 and 1 on all curves. Optimize its positions using $f(m)$ as evaluation.
- Place one control point in each resulting subinterval.

\[
0 < cp2 < cp1 \\
cp1 < cp3 < 1
\]
Determining parameterizations

- Place control point 1 between 0 and 1 on all curves. Optimize its positions using $f(m)$ as evaluation.
- Place one control point in each resulting subinterval.
  \[ 0 < cp2 < cp1 \]
  \[ cp1 < cp3 < 1 \]
- Continue to desired level.
Further optimization

- All control points at once.
- Local intervals.
- Slow but most of the improvement in the beginning.
- Possibly repeated a number of times with new intervals.
Stability

- Collapse of landmarks to one spot can give good evaluation.
- To prevent this, add locked mean shape that cannot collapse.
- This can effect optimality.
Results

Can only correct shapes be represented?

Equally spaced
Positioned by hand
Optimized

+ – 2 std mode 1
+ – 2 std mode 1
+ – 2 mode 1
More results

How well is a correct unseen shape represented?
3D

Closed surfaces

Establishing correspondences

Results

back
Closed surfaces
Algorithm overview

1. **Triangulate the surfaces of the training set.**
2. **Map the triangulations to unit spheres.**
3. **Place initial landmarks on the spheres.**
4. **Optimize positions using genetic algorithm.**
5. **Map the optimal landmark positions back to the original surfaces using the triangulations.**
Triangulating the surfaces

1. Find surface nodes.
2. Find neighbors along voxel edges.
3. Find diagonal neighbors.
4. Place the neighbors in counter clockwise order.
5. Split the square faces into triangles.
Finding surface nodes
Neighbors along voxel edges

There are six possible such neighbors.

Check if they are on the surface.
Neighbors along voxel edges. Cont’d

Check that the path is along the surface.
Diagonal neighbors

There are twelve possible such neighbors.

Check if they are on the surface.
Diagonal neighbors. Cont’d

Check that the path is along the surface.
Mapping triangle corners to unit sphere.

\[ \Theta \]

\[
\begin{align*}
\nabla \vartheta &= 0 \quad (0 < \vartheta < \pi) \\
\vartheta(\text{northpole}) &= 0 \\
\vartheta(\text{southpole}) &= \pi
\end{align*}
\]

\[ A\Theta = b \]
Map triangle corners to unit sphere. Cont’d

\[ \Phi \]

Discontinuity line \( 0/2\pi \).
Remove poles (\( \varphi \) undefined).
Set \( \varphi \) of one node arbitrarily.

\[ A\Phi = b \]
Initial landmarks
Evaluating a set of landmarks

1. **Manipulate landmarks on the spheres.**
2. **Remap the landmarks to the original surfaces.**
3. Build a shape model.
4. Evaluate the model using

\[
\sum \log(\lambda_i + \varepsilon) .
\]

Steps are taken to ensure stability.

**illustration**

**back**
Manipulating the landmark positions

Symmetric theta transformation

- **Place kernels on the sphere.**
- A kernel pushes the landmarks away from the kernel.
- The kernel can be seen as having $\vartheta = 0$. Then each landmark is moved according to

$$f(\vartheta, \alpha, A) = \frac{1}{1 + A} \left( \vartheta + A \arccos \frac{(1 + \alpha^2) \cos \vartheta - 2\alpha}{1 + \alpha^2 - 2\alpha \cos \vartheta} \right).$$

- **Lock $\alpha$ and optimize $A$.**
Placing the kernels in levels
Optimization

\( \alpha \) is determined by the level of the kernel. A for each kernel is to be optimized.

- Optimize A for all kernels on level one.
- Let the kernels have their effect on the landmarks.
- Optimize A for all kernels on the next level
- and so on...
Remapping

Remapping the landmarks from the spheres to the original surfaces.

For each landmark:

1. Find the correct triangle.
2. Find position in triangle.
3. Calculate position on original surfaces.
Finding the correct triangle

- Search-order according to normal of plane defined by triangle.
- Project landmark to the plane.
- Check if the landmark is inside the triangle.
Is the landmark inside the triangle?

Compute the three cross products and check their direction.
Position in triangle

Find coordinates using two sides as unit vectors.

\[ Sp = v \]
Position on original surface

Take the corresponding triangle and use the corresponding sides as unit vectors.

\[ v' = S'p \]
Stability

- Collapse can give good evaluation.
- Add locked mean shape that can not collapse.
- Only use rotation in realignment.
- These things can effect optimality.
Transformations

\[ S_1 \xrightarrow{F_1} S_2 \xrightarrow{F_2} M \]

back
Results

Can only correct shapes be represented?

![Graphs showing initial and optimized shapes with and without module 1 adjustments.](image)
More results

How well can a correct unseen shape be represented?

![Graph showing RMS values for different numbers of modes. The graph compares initial and optimized results.](image)

RMS

number of modes

back
Thank you

Anders Ericsson
and
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